

# Transformations of Functions~Effect on x-values

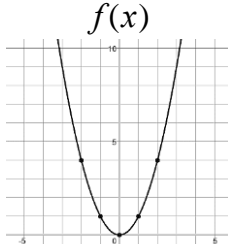
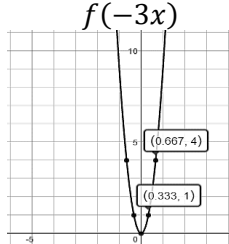
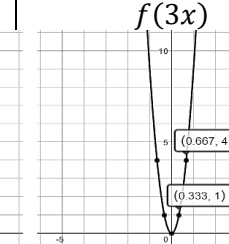
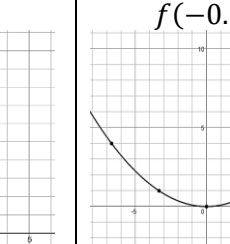
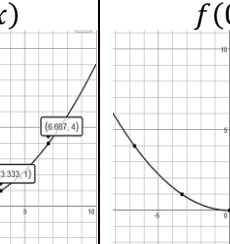
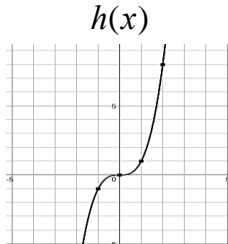
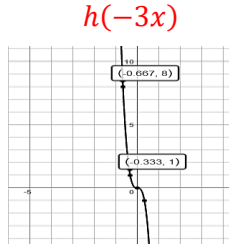
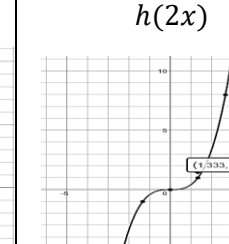
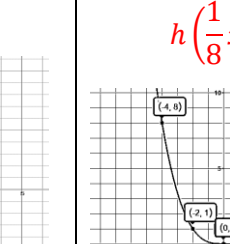
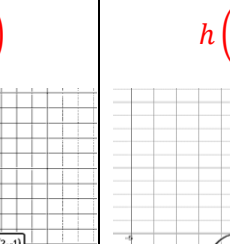
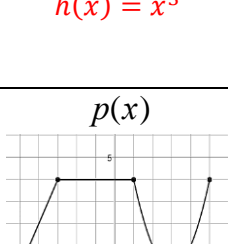
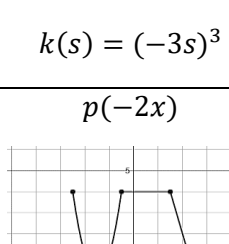
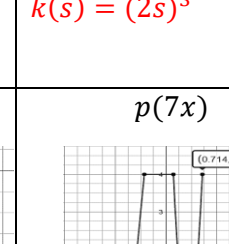
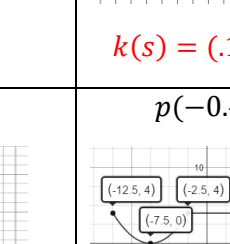
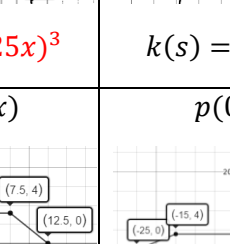
7.3

Secondary Math II Notes

**OBJECTIVE:** Determine the effect of  $k$  on the original function  $f(x)$  if it were replaced with either  $f(kx)$  or  $f(x+k)$  where  $k$  is a real number.

## The effect of $f(kx)$

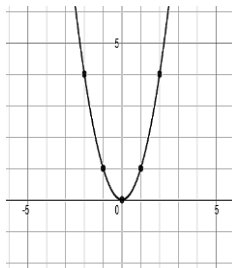
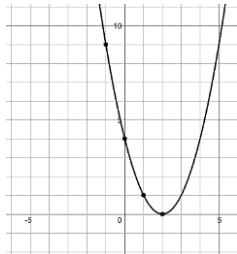
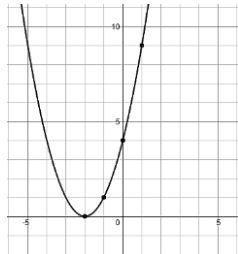
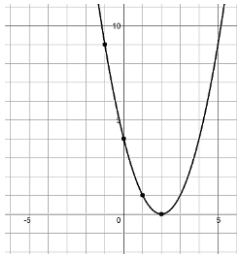
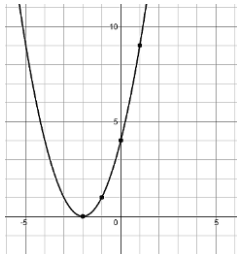
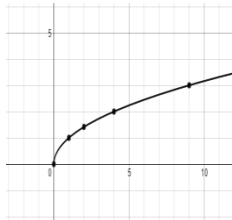
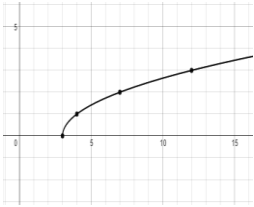
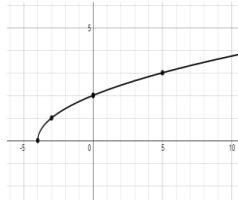
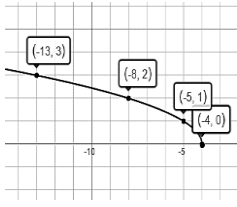
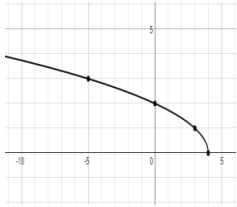
for  $f(x)$  play <https://www.desmos.com/calculator/nqxx2mhknn> and only focus on "k". Change  $f(x)$  to  $f(x) = x^3$ . Then change  $g(x)$  to  $g(x)=f(kx)$  and watch the transformation. Have students begin to make hypothesis about the effect of the constant "k". Then change the equation to  $f(x) = x$  to verify their hypothesis. Have students then sketch the transformations of  $p(x)$ . The parent function always occurs when  $k=1$

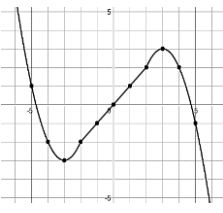
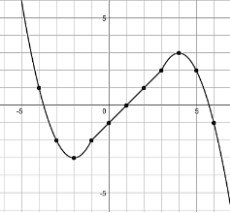
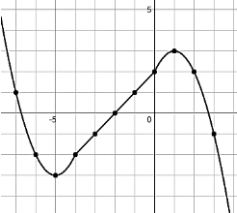
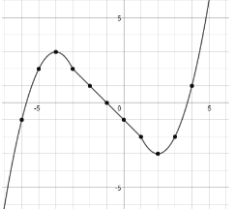

 <p><math>f(x) = x^2</math></p>	 <p><math>k(x) = (-3x)^2</math></p>	 <p><math>k(x) = (3x)^2</math></p>	 <p><math>k(x) = (-.3x)^2</math></p>	 <p><math>k(x) = (.3x)^2</math></p>
 <p><math>h(x) = x^3</math></p>	 <p><math>k(s) = (-3s)^3</math></p>	 <p><math>k(s) = (2s)^3</math></p>	 <p><math>k(s) = (.125s)^3</math></p>	 <p><math>k(s) = (.75s)^3</math></p>
 <p> <math display="block">p(x) = \begin{cases} 2x + 10, &amp; -5 \leq x &lt; -3 \\ 4, &amp; -3 \leq x &lt; 1 \\ (x - 3)^2, &amp; 1 \leq x &lt; 5 \end{cases}</math> </p>	 <p> <math display="block">p(x) = \begin{cases} -4x + 10, &amp; \frac{5}{2} \geq x &gt; \frac{3}{2} \\ 4, &amp; \frac{3}{2} \geq x &gt; \frac{-1}{2} \\ (x - 3)^2, &amp; \frac{-1}{2} \geq x &gt; \frac{-5}{2} \end{cases}</math> </p>	 <p> <math display="block">p(x) = \begin{cases} 14x + 10, &amp; \frac{-5}{7} \leq x &lt; \frac{-3}{7} \\ 4, &amp; \frac{-3}{7} \leq x &lt; \frac{1}{7} \\ (7x - 3)^2, &amp; \frac{1}{7} \leq x &lt; \frac{5}{7} \end{cases}</math> </p>	 <p> <math display="block">p(x) = \begin{cases} -5x + 10, &amp; \frac{25}{2} \geq x &gt; \frac{15}{2} \\ 4, &amp; \frac{15}{2} \geq x &gt; \frac{-5}{2} \\ (-\frac{5}{2}x - 3)^2, &amp; \frac{-5}{2} \geq x &gt; \frac{-25}{2} \end{cases}</math> </p>	 <p> <math display="block">p(x) = \begin{cases} 10x + 10, &amp; -25 \leq x &lt; -15 \\ 4, &amp; -15 \leq x &lt; 5 \\ (5x - 3)^2, &amp; 5 \leq x &lt; 25 \end{cases}</math> </p>

<p>Answer the following four questions for each column</p> <ol style="list-style-type: none"> <li>1. What type of transformation occurred? Be specific.</li> <li>2. How did this transformation affect the x-values?</li> <li>3. How did this transformation affect the y-values?</li> <li>4. Did it affect the domain or the range? Explain why.</li> </ol>	<ol style="list-style-type: none"> <li>1. There was a reflection over the y-axis because <math>k &lt; 0</math> and a horizontal compression by a factor of <math>k</math> units because <math> k  &gt; 1</math>.</li> <li>2. The x-values were compressed by a factor of <math>k</math> units</li> <li>3. The y-values were not affected</li> <li>4. The domain was affected because the x-values were changed.</li> </ol>	<ol style="list-style-type: none"> <li>1. There is a horizontal compression by a factor of <math>k</math> units because <math> k  &gt; 1</math>.</li> <li>2. The x-values were compressed by a factor of <math>k</math> units</li> <li>3. The y-values were not affected</li> <li>4. The domain was affected because the x-values were changed.</li> </ol>	<ol style="list-style-type: none"> <li>1. There was a reflection over the y-axis because <math>k &lt; 0</math> and a horizontal stretch by a factor of <math>k</math> units because <math>0 &lt;  k  &lt; 1</math>.</li> <li>2. The x-values were stretched by a factor of <math>k</math> units</li> <li>3. The y-values were not affected</li> <li>4. The domain was affected because the x-values were changed.</li> </ol>	<ol style="list-style-type: none"> <li>1. There was a horizontal stretch by a factor of <math>k</math> units because <math>0 &lt;  k  &lt; 1</math></li> <li>2. The x-values were stretched by a factor of <math>k</math> units</li> <li>3. The y-values were not affected</li> <li>4. The domain was affected because the x-values were changed.</li> </ol>
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**The effect of  $f(x + k)$**

for  $f(x)$  play <https://www.desmos.com/calculator/nqxx2mhknn> and only focus on "k". Change  $f(x)$  to  $f(x) = x^2$ . Then change  $g(x)$  to  $g(x) = f(x+k)$  and watch the transformation. Have students begin to make hypothesis about the effect of the constant "k". Then change the equation to  $f(x) = x^3$  to verify their hypothesis. Have students then sketch the transformations of  $p(x)$ . The parent function always occurs when  $k=1$

<p style="text-align: center;"><math>f(x)</math></p>  <p style="text-align: center;"><math>f(x) = x^2</math></p>	<p style="text-align: center;"><math>f(x - 2)</math></p>  <p style="text-align: center;"><math>f(a) = (a - 2)^2</math></p>	<p style="text-align: center;"><math>f(x + 2)</math></p>  <p style="text-align: center;"><math>g(r) = (r + 2)^2</math></p>	<p style="text-align: center;"><math>f(-x - 2)</math></p>  <p style="text-align: center;"><math>k(s) = (-s - 2)^2</math></p>	<p style="text-align: center;"><math>f(-x + 2)</math></p>  <p style="text-align: center;"><math>g(x) = (-x + 2)^2</math></p>
<p style="text-align: center;"><math>h(x)</math></p>  <p style="text-align: center;"><math>h(x) = \sqrt{x}</math></p>	<p style="text-align: center;"><math>h(x - 3)</math></p>  <p style="text-align: center;"><math>k(r) = \sqrt{r - 3}</math></p>	<p style="text-align: center;"><math>h(x + 4)</math></p>  <p style="text-align: center;"><math>k(r) = \sqrt{r + 4}</math></p>	<p style="text-align: center;"><math>h(-x - 4)</math></p>  <p style="text-align: center;"><math>l(s) = \sqrt{-s - 4}</math></p>	<p style="text-align: center;"><math>h(-x + 4)</math></p>  <p style="text-align: center;"><math>k(r) = \sqrt{-r + 4}</math></p>

<p style="text-align: center;"><math>p(x)</math></p>  $p(x) = \begin{cases} (x+3)^2 - 3, & x \leq -2 \\ x, & -2 < x \leq 2 \\ -(x-3)^2 + 3, & x > 2 \end{cases}$	<p style="text-align: center;"><math>p(x-1)</math></p>  $k(x) = \begin{cases} (x+2)^2 - 3, & x \leq -1 \\ x-1, & -1 < x \leq 3 \\ -(x-4)^2 + 3, & x > 3 \end{cases}$	<p style="text-align: center;"><math>p(x+2)</math></p>  $g(x) = \begin{cases} (x+5)^2 - 3, & x \leq -4 \\ x+2, & -4 < x \leq 0 \\ -(x-1)^2 + 3, & x > 0 \end{cases}$	<p style="text-align: center;"><math>p(-x-1)</math></p>  $r(x) = \begin{cases} (-x+2)^2 - 3, & x \geq 1 \\ -x-1, & 1 > x \geq -3 \\ -(-x-4)^2 + 3, & x < -3 \end{cases}$	<p style="text-align: center;"><math>p(-x+2)</math></p>  $k(x) = \begin{cases} (-x+5)^2 - 3, & x \geq 4 \\ -x+2, & 4 > x \geq 0 \\ -(-x-1)^2 + 3, & x < 0 \end{cases}$
<p>Answer the following four questions for each column</p> <ol style="list-style-type: none"> <li>1. What type of transformation occurred? Be specific.</li> <li>2. How did this transformation affect the x-values?</li> <li>3. How did this transformation affect the y-values?</li> <li>4. Did it affect the domain or the range? Explain why.</li> </ol> <p>Emphasize the order of the horizontal reflection and horizontal shift</p>	<ol style="list-style-type: none"> <li>1. There was horizontal shift to the right k units because <math>k &lt; 0</math>.</li> <li>2. The x-values were shifted to the right by k units</li> <li>3. The y-values were not affected</li> <li>4. The domain was affected because the x-values were changed.</li> </ol>	<ol style="list-style-type: none"> <li>1. There was horizontal shift to the left k units because <math>k &gt; 0</math>.</li> <li>2. The x-values were shifted to the left by k units</li> <li>3. The y-values were not affected</li> <li>4. The domain was affected because the x-values were changed.</li> </ol>	<ol style="list-style-type: none"> <li>1. There was horizontal shift to the right k units because <math>k &lt; 0</math> followed by a reflection across the y-axis.</li> <li>2. The x-values were shifted to the right by k units and then multiplied by -1.</li> <li>3. The y-values were not affected</li> <li>4. The domain was affected because the x-values were changed.</li> </ol>	<ol style="list-style-type: none"> <li>1. There was horizontal shift to the left k units because <math>k &gt; 0</math> followed by a reflection across the y-axis.</li> <li>2. The x-values were shifted to the left by k units and then multiplied by -1.</li> <li>3. The y-values were not affected</li> <li>4. The domain was affected because the x-values were changed.</li> </ol>