Secondary Math II Notes
OBJECTIVE: Compose functions graphically and algebraically. Write composition of functions to model scenarios.
Composition of Functions: The composition of two functions is written as $\left(f^{\circ} g\right)(x)$. The function $\left(f^{\circ} g\right)(x)=f(g(x))$.

## Composition of functions graphically

On a graphing calculator, graph
$f(x)=-x-1$ and $g(x)=2 x-3$.
Fill in the table below

| $X$ | $f(x)$ | $g(x)$ |
| :--- | :--- | :--- |
| -6 | 5 | -9 |
| -5 | 4 | -8 |
| -4 | 3 | -7 |
| -3 | 2 | -6 |
| -2 | 1 | -5 |
| -1 | 0 | -4 |
| 0 | -1 | -3 |
| 1 | -2 | -2 |
| 2 | -3 | -1 |
| 3 | -4 | 0 |
| 4 | -5 | 1 |
| 5 | -6 | 2 |
| 6 | -7 | 3 |

On a graphing calculator, graph
$f(x)=x-2$ and $g(x)=-x+1$.
Fill in the table below

| $X$ | $f(x)$ | $g(x)$ |
| :--- | :--- | :--- |
| -6 | -8 | 7 |
| -5 | -7 | 6 |
| -4 | -6 | 5 |
| -3 | -5 | 4 |
| -2 | -4 | 3 |
| -1 | -3 | 2 |
| 0 | -2 | 1 |
| 1 | -1 | 0 |
| 2 | 0 | -1 |
| 3 | 1 | -2 |
| 4 | 2 | -3 |
| 5 | 3 | -4 |
| 6 | 4 | -5 |

Graph the function
$\left(f^{\circ} g\right)(x)$ using the functions to the

| $X$ | $\left(f^{\circ} g\right)(x)$ |
| :--- | :--- |
| -3 | 5 |
| -2 | 4 |
| -1 | 3 |
| 0 | 2 |
| 1 | 1 |

right.
Graph the function
( $g^{\circ} f$ ) ( $x$ ) using the functions to the right.

| $X$ | $\left(g^{\circ} f\right)(x)$ |
| :--- | :--- |
| -3 | -1 |
| -2 | -2 |
| -1 | -3 |
| 0 | -4 |
| 1 | -5 |

Graph the function $g(g(x))$ using the functions to the

| $X$ | $g(g(x))$ |
| :--- | :--- |
| -3 | -3 |
| -2 | -2 |
| -1 | -1 |
| 0 | 0 |
| 1 | 1 |

right.
Graph the function
$f(f(x))$ using the functions to the right.

| $X$ | $f(f(x))$ |
| :--- | :--- |
| -3 | -7 |
| -2 | -6 |
| -1 | -5 |
| 0 | -4 |
| 1 | -3 |













| Composition of functions algebraically |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Use the following functions for the problems below$f(x)=-5 x, \quad g(x)=2 x-1, \quad h(x)=-x^{2}-4 x-1, \quad k(x)=x^{2}-x$ |  |  |  |  |  |  |
| $\begin{aligned} &\left(f^{\circ}\right. \\ &\left(f^{\circ} g\right)(x) \\ &= f(2 x \\ &=-5(2 x \\ &=-10 x \end{aligned}$ | $\begin{aligned} & g)(x) \\ & =f(g \\ & -1) \\ & -1) \\ & +5 \end{aligned}$ |  | $\begin{aligned} & h(f(t) \\ & =-( \\ & -1 \\ & =-2 \end{aligned}$ | $\begin{aligned} & h(f(t)) \\ & )=h(-5 t) \\ & -5 t)^{2}-4(-5 t) \\ & t^{2}+20 t-1 \end{aligned}$ | $\begin{aligned} & g(f(3 r)) \\ & g(-5(3 r))= g(-15 r) \\ &=2(-15 r) \\ &-1 \\ &=-30 r-1 \end{aligned}$ | $\begin{gathered} f(h(4)) \\ f\left(-(4)^{2}-4(4)-1\right) \\ =f(-33)=165 \end{gathered}$ |
| $\begin{array}{r} \left(g^{\mathrm{o}}\right. \\ \left(g^{\circ} k\right)(x) \\ =g \\ =2\left(x^{2}\right. \\ =2 x^{2} \end{array}$ | $\begin{aligned} & k)(x) \\ & =g(k \\ & \left.x^{2}-x\right) \\ & -x)- \\ & -2 x- \end{aligned}$ |  | $\begin{gathered} g(g(a) \\ = \\ =4 a \end{gathered}$ | $\begin{aligned} & g(g(a)) \\ & )=g(2 a-1) \\ & (2 a-1)-1 \\ & 3 \end{aligned}$ | $\begin{gathered} \left(k^{\circ} f\right)(2 n) \\ \left(k^{\circ} f\right)(2 n)=k(f(2 n)) \\ =k(-10 n) \\ =(-10 n)^{2}-(-10 n) \\ =100 n^{2}+10 n \end{gathered}$ | $\begin{aligned} & f(f(3)) \\ = & f(-5(3)) \\ = & f(-15) \\ = & -5(-15) \\ = & 75 \end{aligned}$ |
| Application of Composition of functions |  |  |  |  |  |  |
| You work forty hours a week at a furniture store. You receive a $\$ 220$ weekly salary, plus a 3\% commission on sales over $\$ 5000$. Assume that you sell enough this week to get the commission. <br> a) Write the commission sales as a function of total sales. $S(x)=x-5000$ <br> b) Write the commission as a function of the commission sales. $C(r)=0.03 r$ <br> c) Write the commission as a function of the total sales. The commission is represented by $\left(c^{\circ} S\right)(x)=0.03(x-$ 5000) $=0.03 x-150$ <br> The table is representing how the formula would work. |  |  |  |  | Maurice's is having an end of season clearance sale. In the mail you receive a coupon for $\$ 5$ off of a pair of jeans. When you arrive at the store, you find that all jeans are $25 \%$ off. <br> a) Write the cost of the jeans as a function if you used the coupon. $f(x)=x-5$ <br> b) Write the cost of the jeans as a function if you used the discount of $25 \% . g(x)=.75 x$ <br> Option 1- you use the $\$ 5$ coupon first and then you use the $25 \%$ off on the remaining amount. <br> Option 2- you use the $25 \%$ off first and then you use the $\$ 5$ off on the remaining amount. <br> c) Write a function that would represent option 1 <br> d) Write a function that would represent option 2 <br> e) Which option saves you the most money? <br> Option 1 would be $g(f(x))=.75(x-5)=.75 x-3.75$ <br> Option 2 would be $f(g(x))=.75 x-5$ <br> Since they both have the same coefficient for the $x$ variable and option 2 subtracts 5 instead of 3.75 , option 2 will save you the most money. |  |



