Average Rate of Change { 5.6 }

Secondary Math II Notes

OBJECTIVE: Find the slope of linear functions. Use the same strategy to find the average rate of change for non-linear functions. Do so using points, tables, and graphs.

Review of Slope

Slope

A number that describes how steep a line is. A line with a positive slope goes up to the right. A line with a negative slope goes down to the right. Slope is the difference in y-values divided by the difference in x-values.

$$slope = \frac{y_a - y_b}{x_a - x_b}$$

Find the slope for each of the linear functions described below.

$$slope = \frac{y_a - y_b}{x_a - x_b} = \frac{2-0}{0-2} = \frac{2}{-2} = -1$$

X	2	3	4	5	6
Y	-12	-8	-4	0	4

$$slope = \frac{y_a - y_b}{x_a - x_b} = \frac{0-(-12)}{5-2} = \frac{12}{3} = 4$$

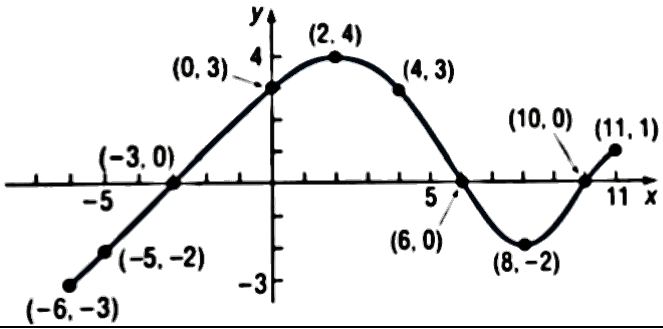
Consider the line that passes through the following points:

(1,17) & (4,15)

$$slope = \frac{y_a - y_b}{x_a - x_b} = \frac{17-15}{1-4} = \frac{2}{-3} = -\frac{2}{3}$$

Average Rate of Change From a Graph

The average rate of change is simply the slope of the straight line that passes through two particular points on a function, even if the function itself is not a straight line.



Instructions:

When you are given an interval, find the points at the each end of that interval. Then use the slope formula to find the slope of the straight line between them.

What is the average rate of change over the interval (-3, 2)? Use the points (-3, 0) and (2, 4).

$$slope = \frac{y_a - y_b}{x_a - x_b} = \frac{0-4}{-3-2} = \frac{-4}{-5} = \frac{4}{5}$$

What is the average rate of change over the interval (-3, 6)? Use the points (-3, 0) and (6, 0).

$$slope = \frac{y_a - y_b}{x_a - x_b} = \frac{0-0}{-3-6} = \frac{0}{-9} = 0$$

What is the average rate of change over the interval (2, 8)? Use the points (2, 4) and (8, -2).

$$slope = \frac{y_a - y_b}{x_a - x_b} = \frac{4-(-2)}{2-8} = \frac{6}{-6} = -1$$

What is the average rate of change over the interval (-6, 8)? Use the points (-6, -3) and (8, -2).

$$slope = \frac{y_a - y_b}{x_a - x_b} = \frac{-3-(-2)}{-6-8} = \frac{-1}{-14} = \frac{1}{14}$$

Average Rate of Change From a Table

The table below represents the average price for a ticket to a movie theater in North America from 1987 to 2009.

Year	1987	1991	1995	1999	2003	2007	2009
Price ($)	3.91	4.21	4.35	5.06	6.03	6.88	7.50

What is the average rate of change from 1991 to 2007?

Use the points (1991, 4.21) and (2007, 6.88).

$$slope = \frac{y_a - y_b}{x_a - x_b} = \frac{6.88 - 4.21}{2007 - 1991} = \frac{2.67}{16} = .166875$$

What is the average rate of change from 1995 to 2009?

Use the points (1995, 4.35) and (2009, 7.50).

$$slope = \frac{y_a - y_b}{x_a - x_b} = \frac{7.50 - 4.35}{2009 - 1995} = \frac{3.15}{14} = .225$$

Which of the following statements is true?

a. *The average rate of change from 1991 to 2007 was greater than the average rate of change from 1995 to 2009.*
b. *The average rate of change from 1991 to 2007 was less than the average rate of change from 1995 to 2009.*
c. *The average rate of change from 1991 to 2007 was the same as the average rate of change from 1995 to 2009.*

Average Rate of Change From a Function

Instructions: When you are given an interval, find the points at the each end of that interval by plugging the x-values into the function. Then use the slope formula to find the slope of the straight line between the two points that you have found..

Find the average rate of change over the interval (2, 6) for the function

$$h(x) = \frac{1}{2}x + 6$$

$$h(2) = \frac{1}{2} \cdot 2 + 6 = 1 + 6 = 7$$

$$(2,7)$$

$$h(6) = \frac{1}{2} \cdot 6 + 6 = 3 + 6 = 9$$

$$(6,9)$$

$$slope = \frac{y_a - y_b}{x_a - x_b} = \frac{9 - 7}{6 - 2} = \frac{2}{4} = \frac{1}{2}$$

Find the average rate of change over the interval (0, 4) for the function

$$g(x) = 4(2)^x$$

$$g(0) = 4(2)^0 = 4 \cdot 1 = 4$$

$$(0,4)$$

$$g(4) = 4(2)^4 = 4 \cdot 16 = 64$$

$$(4,64)$$

$$slope = \frac{y_a - y_b}{x_a - x_b} = \frac{64 - 4}{4 - 0} = \frac{60}{4} = 15$$