Key Features of Graphs {5.5}

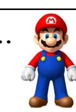
Secondary Math II Notes

OBJECTIVE: Identify the key features of a graph including: maxima, minima, y-intercept, x-intercepts, and end behavior. Identify intervals of where a function is positive, negative, increasing, decreasing, and constant. Express these intervals in interval notation.

Important *Points* on a Graph

X-intercepts	X-intercepts are the points that lie on the x-axis. Functions may have more than one x-intercept. The y-value for an x-intercept is always 0.	(x-intercept, 0) This is an ordered pair.
Y-intercept	Y-intercepts are the points that lie on the y-axis. Functions will not have more than one y-intercept. The x-value for a y-intercept is always 0.	(0, y-intercept) This is an ordered pair.
Maxima (Plural for maximum)	A relative maximum is a point that is higher than the other points directly to the right or left.	(x,y) y tells us what the maximum is and x tells us where it occurs.
Minima (Plural for minimum)	A relative minimum is a point that is lower than the other points directly to the right or left.	(x,y) y tells us what the minimum is and x tells us where it occurs.

Important *Intervals* on a Graph

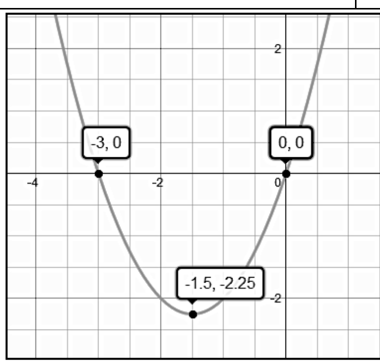
Type of interval:	How we write it:	What it means for Mario.... 
Increasing: A portion of a function is said to be increasing when the y-values INCREASE as the x-values increase.	(beginning x-value, ending x value) This interval only uses x-values. Do not EVER use y-values.	As Mario runs from left to right he is running uphill.
Decreasing: A portion of a function is said to be decreasing when the y-values DECREASE as the x-values increase.	(beginning x-value, ending x value) This interval only uses x-values. Do not EVER use y-values.	As Mario runs from left to right he is running downhill.
Constant: A portion of a function is said to be constant when the y-values remain the same as the x-values increase.	(beginning x-value, ending x value) This interval only uses x-values. Do not EVER use y-values	As Mario runs from left to right he is running on flat ground.
Positive: A portion of the graph is positive if the y-values are positive.	(beginning x-value, ending x value) This interval only uses x-values. Do not EVER use y-values.	If the x-axis represented the surface of a body of water, Mario would be under water here.
Negative: A portion of the graph is negative if the y-values are negative.	(beginning x-value, ending x value) This interval only uses x-values. Do not EVER use y-values.	If the x-axis represented the surface of a body of water, Mario would be above water here.

A note about parentheses and brackets: We will always use parentheses for intervals that are increasing, decreasing, and constant, because a single point cannot be defined as any of those things. A single point CAN be defined as positive or negative though, so there may be times when we use brackets for positive and negative intervals.

End Behavior

An explanation of how the y-values behave when the x-value approach positive and negative infinity (at the "ends" of the graph).

End Behavior	Examples
As $x \to -\infty$, $y \to -\infty$ *(as x approaches negative infinity, y approaches negative infinity)* *As $x \to +\infty$, $y \to +\infty$*	This is like saying, "As we go left, the y-values go down, and as we go right the y-values go up."
As $x \to -\infty$, $y \to +\infty$ *As $x \to +\infty$, $y \to -\infty$*	This is like saying, "As we go left, the y-values go up, and as we go right the y-values go down."
As $x \to -\infty$, $y \to -\infty$ *As $x \to +\infty$, $y \to -\infty$*	This is like saying, "As we go left, the y-values go down, and as we go right the y-values go down."
As $x \to -\infty$, $y \to +\infty$ *As $x \to +\infty$, $y \to +\infty$*	This is like saying, "As we go left, the y-values go up, and as we go right the y-values go up."
As $x \to -\infty$, $y \to -2$ *As $x \to +\infty$, $y \to +\infty$*	This is like saying, "As we go left, the y-values get closer and closer to -2, and as we go right the y-values go up."



x-intercept(s): (-3,0) and (0,0)

y-intercept: (0,0)

maxima: none

minima: (-1.5,-2.25)

interval of increasing: (-1.5, ∞)

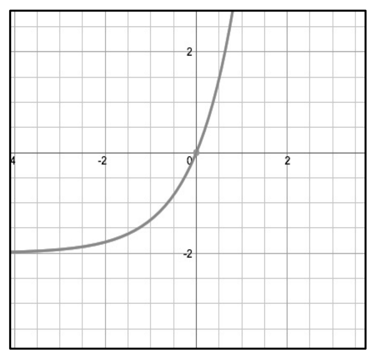
interval of decreasing: (-∞, -1.5)

interval of constant: none

interval of positive: (-∞,-3) U (0, ∞)

interval of negative: (-3,0)

end behavior: As $x \to -\infty$, $y \to +\infty$. As $x \to +\infty$, $y \to +\infty$.



x-intercept(s): (0,0)

y-intercept: (0,0)

maxima: none

minima: none

interval of increasing: (-∞,∞)

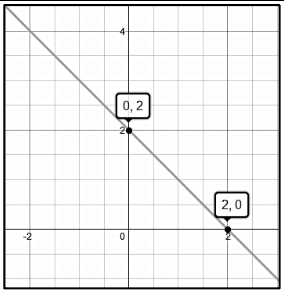
interval of decreasing: none

interval of constant: none

interval of positive: (0, ∞)

interval of negative: (-∞,0)

end behavior: As $x \to -\infty$, $y \to -2$. As $x \to +\infty$, $y \to +\infty$.



x-intercept(s): (2,0)

y-intercept: (0,2)

maxima: none

minima: none

interval of increasing: none

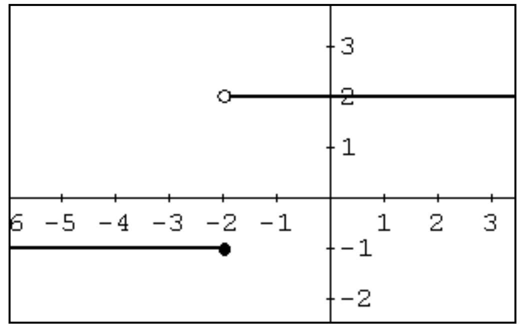
interval of decreasing: (-∞,∞)

interval of constant: none

interval of positive: (-∞, 2)

interval of negative: (2, ∞)

end behavior: As x→ -∞, y→+∞. As x→ +∞, y→-∞.



x-intercept(s): none

y-intercept: (0,2)

maxima: none

minima: none

interval of increasing: none

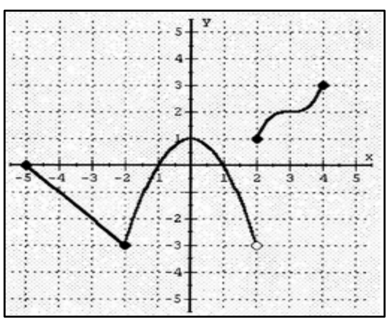
interval of decreasing: none

interval of constant: (-∞, -2) U (-2, ∞)

interval of positive: (-2, ∞)

interval of negative: (-∞, -2)

end behavior: As x→ -∞, y→-1. As x→ +∞, y→2.



x-intercept(s): (-5, 0) and (-1, 0) and (1, 0)

y-intercept: (0,1)

maxima: (0, 1)

minima: (-2, -3)

interval of increasing: (-2, 0) U (2, 4)

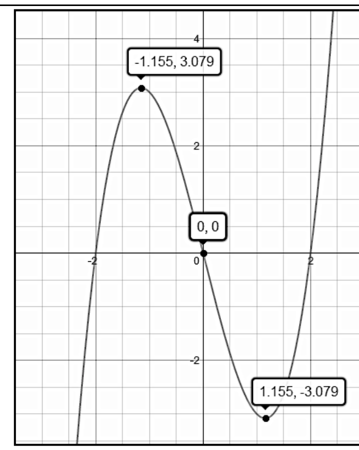
interval of decreasing: (-5, -2) U (0, 2)

interval of constant: none

interval of positive: {-5} U (-1, 1) U [2, 4]

interval of negative: (-5, -1) U (1, 2)

end behavior: not applicable



x-intercept(s): (-2, 0) and (0, 0) and (2, 0)

y-intercept: (0, 0)

maxima: (-1.155, 3.079)

minima: (1.155, -3.079)

interval of increasing: (-∞, -1.155) U (1.155, ∞)

interval of decreasing: (-1.155, 1.155)

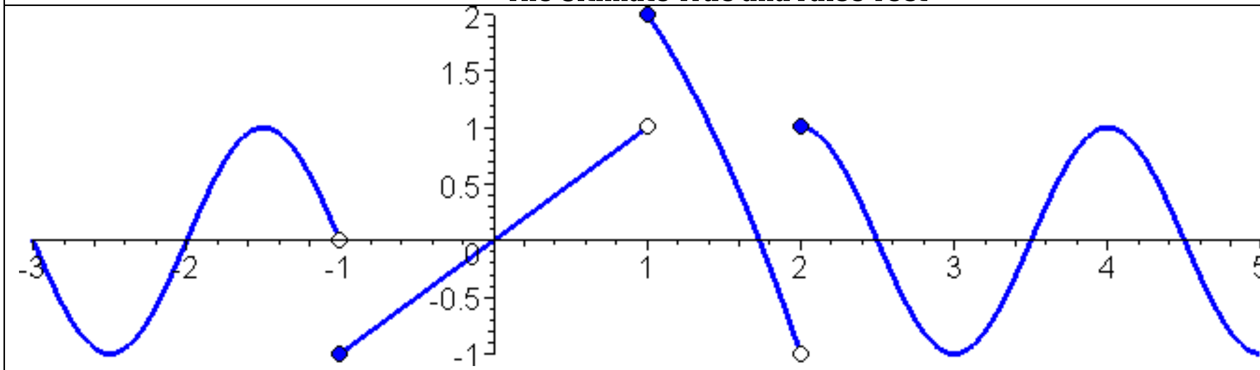
interval of constant: none

interval of positive: (-2, 0) U (2, ∞)

interval of negative: (-∞, -2) U (0, 2)

end behavior: As x→ -∞, y→-∞. As x→ +∞, y→+∞.

The Ultimate True and False Test



1._______ This function has a maximum at (2,1). F

2._______ This function is positive on the interval (0,5). F

3._______ This function is decreasing on the interval (2,3). T

4._______ This function is increasing on the interval [-1,1). F

5._______ This function has a minimum at (2,-1). F

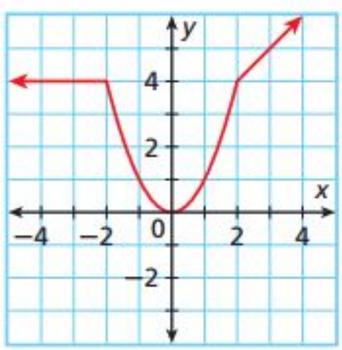
6._______ This function has an x-intercept at (0,0). T

7._______ This function is negative on the interval (4,5). F

8._______ This function is increasing on the interval (3,4). T

9._______ This function has a minimum at (3,-1). T

10._______ This function has an x-intercept at (0,-2). F



11.______ This function is positive on the interval (-2, ∞). F

12._______ This function is constant on the interval (-∞,-2). T

13._______ This function is negative on the interval (-∞,0). F

14._______ This function has a minimum at [0,0]. F

15._______ *As* $x \to +\infty$, $y \to +\infty$. T

16._______ This function is increasing on the interval (0, ∞). T

17._______ This function is decreasing on the interval (4,0). F

18._______ This function has a maximum at (-2,4). F