

Trigonometric Ratios

Secondary Math II Notes

{ 9.2 }

OBJECTIVE: Define sine, cosine, and tangent. Determine trigonometric ratios for triangles.

Exploration

In the space below, use a ruler and protractor to draw a right triangle that also has a 60-degree angle. Measure each side of your triangle as accurately as you can in centimeters. Using the 60-degree angle as the angle of reference list the measure for each of the following:

****Students draw their triangle. Answers will vary.**

Length of the **adjacent** side: _____

Length of the **opposite** side: _____

Length of the **hypotenuse**: _____

Create the following ratios (give the decimal) using your measurements from above:

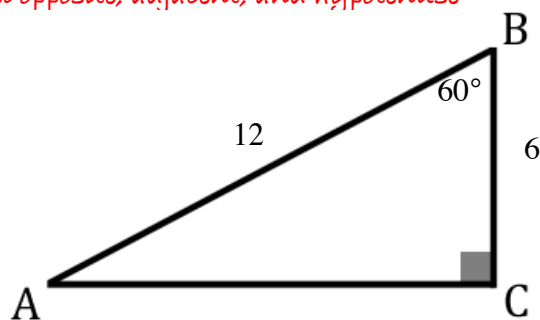
$$\frac{\text{opposite side}}{\text{hypotenuse}} =$$

$$\frac{\text{adjacent side}}{\text{hypotenuse}} =$$

$$\frac{\text{opposite side}}{\text{adjacent side}} =$$

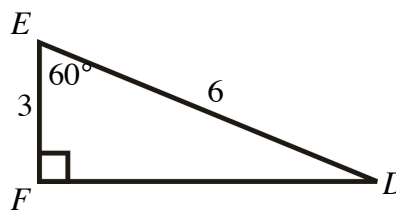
Below are two right triangles. Each has a 60-degree angle and is similar to the triangle that you created above. These triangles have been created and measured by a computer to avoid any human error. Use the Pythagorean Theorem to find the length of each missing side. Find the ratios below and compare them to the ratios found for your triangle.

Label opposite, adjacent, and hypotenuse



$$12^2 - 6^2 = 108 = 6\sqrt{3}$$

Label opposite, adjacent, and hypotenuse



$$6^2 - 3^2 = 27 = 3\sqrt{3}$$

List the ratios for $\triangle ABC$ using angle A as the angle of reference.

List the ratios for $\triangle DEF$ using angle E as the angle of reference.

$$\frac{\text{opposite side}}{\text{hypotenuse}} =$$

$$\frac{\text{adjacent side}}{\text{hypotenuse}} =$$

$$\frac{\text{opposite side}}{\text{adjacent side}} =$$

$$\frac{\text{opposite side}}{\text{hypotenuse}} =$$

$$\frac{\text{adjacent side}}{\text{hypotenuse}} =$$

$$\frac{\text{opposite side}}{\text{adjacent side}} =$$

$$\frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2} = 0.87$$

$$\frac{6}{12} = \frac{1}{2} = 0.5$$

$$\frac{6\sqrt{3}}{6} = \sqrt{3} = 1.73$$

$$\frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2} = 0.87$$

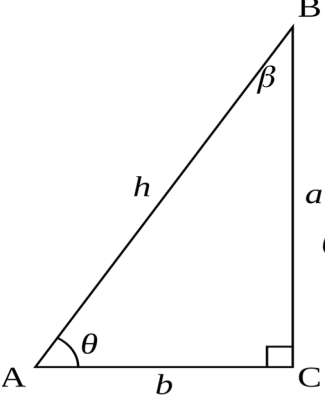
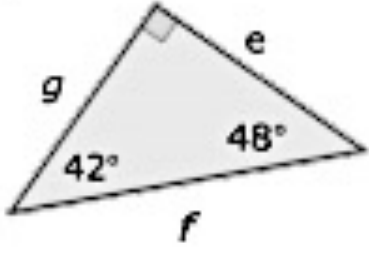
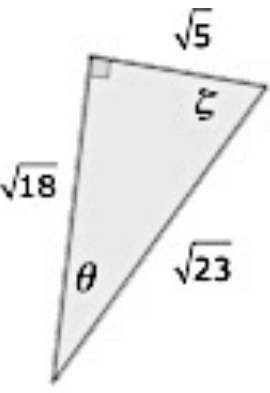
$$\frac{3}{6} = \frac{1}{2} = 0.5$$

$$\frac{3\sqrt{3}}{3} = \sqrt{3} = 1.73$$

Trigonometric Ratios

| Sine | Cosine | Tangent |
|---|---|--|
| $\sin(\theta) = \frac{\text{opposite side}}{\text{hypotenuse}}$ | $\cos(\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}}$ | $\tan(\theta) = \frac{\text{opposite side}}{\text{adjacent side}}$ |

Use the following triangles to find the trig ratios below.

| | | |
|---|--|---|
|  | $\sin(\theta) = \frac{a}{h}$ $\cos(\theta) = \frac{b}{h}$ $\tan(\theta) = \frac{a}{b}$ | $\sin(\beta) = \frac{b}{h}$ $\cos(\beta) = \frac{a}{h}$ $\tan(\beta) = \frac{b}{a}$ |
|  | $\sin(42^\circ) = \frac{e}{f}$ $\cos(42^\circ) = \frac{g}{f}$ $\tan(42^\circ) = \frac{e}{g}$ | $\sin(48^\circ) = \frac{g}{f}$ $\cos(48^\circ) = \frac{e}{f}$ $\tan(48^\circ) = \frac{g}{e}$ |
|  | $\sin(\theta) = \frac{\sqrt{5}}{\sqrt{23}}$ $\cos(\theta) = \frac{\sqrt{18}}{\sqrt{23}}$ $\tan(\theta) = \frac{\sqrt{5}}{\sqrt{18}}$ | $\sin(\zeta) = \frac{\sqrt{18}}{\sqrt{23}}$ $\cos(\zeta) = \frac{\sqrt{5}}{\sqrt{23}}$ $\tan(\zeta) = \frac{\sqrt{18}}{\sqrt{5}}$ |

What do you notice about the relationship between sine, cosine, and tangent for each reference triangle?

Sine for a reference angle is equal to cosine of the complementary angle. Tangent is the reciprocal for the opposite reference angle.