Pythagorean Theorem {9.1}

Secondary Math II Notes

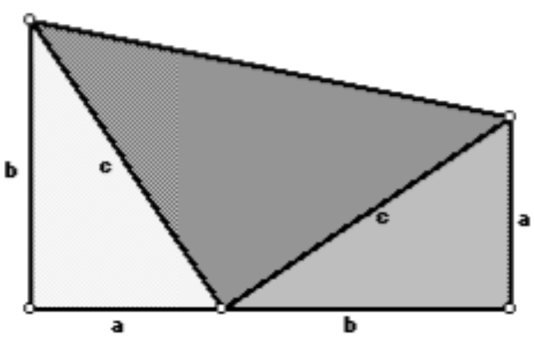
OBJECTIVE: Prove the Pythagorean Theorem and use it to find side lengths of triangles.

Type of Proofs:

Two Column	Algebraic	Proofs without words
Statement | Reason	Using numbers and equality statements (i.e. solving an equation)	Pictures or diagrams that help the reader see why a particular mathematical statement may be true

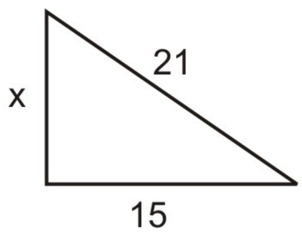
Pythagorean Theorem: In a right triangle, one leg squared plus the other leg squared equals the hypotenuse squared, $a^2 + b^2 = c^2$.

President Garfield's Proof:

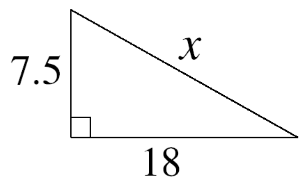


Statements	Reason
$A = \frac{b_1 + b_2}{2} \cdot h$	Area of a trapezoid
$h = a + b,\ b_1 = a,\ b_2 = b$	Based on the diagram to the right.
$A = \frac{a+b}{2} \cdot (a+b)$	Substituting Values
$A = \frac{a^2 + 2ab + b^2}{2} = \frac{1}{2}\left(a^2 + 2ab + b^2\right)$	Distribution and simplifying
$A_1 = \frac{1}{2} \cdot ba,\ A_2 = \frac{1}{2} \cdot c^2,\ A_3 = \frac{1}{2} \cdot ab$	Areas of triangles in diagram
$A = \frac{1}{2} \cdot ba + \frac{1}{2} \cdot c^2 + \frac{1}{2} \cdot ab$	Sum of triangle areas
$A = \frac{1}{2}\left(ba + c^2 + ab\right)$	Factor out 1/2
$A = \frac{1}{2}\left(2ab + c^2\right)$	Combine like terms
$\frac{1}{2}\left(a^2 + 2ab + b^2\right) = \frac{1}{2}\left(2ab + c^2\right)$	Since the areas are equivalent
$a^2 + b^2 = c^2$	Multiply by 2 and subtract 2ab from both sides

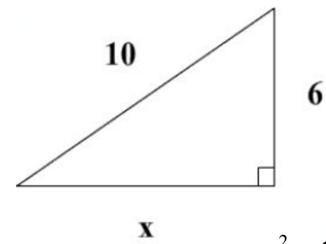
Using the Pythagorean Theorem



$$x^2 + 15^2 = 21^2$$
$$21^2 - 15^2 = x^2$$
$$216 = x^2$$
$$6\sqrt{6} = x$$



$$7.5^2 + 18^2 = x^2$$
$$380.25 = x^2$$
$$19.5 = x$$



$$x^2 + 6^2 = 10^2$$
$$10^2 - 6^2 = x^2$$
$$64 = x^2$$
$$8 = x$$

A right triangle has side lengths a, b, and c, where c is the hypotenuse. Solve for the missing side.

$a = \sqrt{3}$

$b = 7$

$c =$

$$\left(\sqrt{3}\right)^2 + 7^2 = c^2$$
$$52 = c^2$$
$$2\sqrt{13} = c$$

A right triangle has side lengths a, b, and c, where c is the hypotenuse. Solve for the missing side.

$a = \sqrt{5}$

$b =$

$c = \sqrt{21}$

$$\left(\sqrt{21}\right)^2 - \left(\sqrt{5}\right)^2 = b^2$$
$$21 - 5 = b^2$$
$$16 = b^2$$
$$4 = b$$

A right triangle has side lengths a, b, and c, where c is the hypotenuse. Solve for the missing side.

$a =$

$b = 16$

$c = 20$

$$20^2 - 16^2 = a^2$$
$$144 = a^2$$
$$12 = a$$

Classifying Triangles

If $a^2 + b^2 > c^2$, then the triangle is ______Acute___________.

If $a^2 + b^2 < c^2$, then the triangle is _________Obtuse________.

If $a^2 + b^2 = c^2$, then the triangle is ________Right_________.

Determine whether the given side lengths would be create an acute, obtuse, or right triangle

Side Lengths: 12, 15, 9

$$9^2 + 12^2 \_\_ 15^2$$
$$81 + 144 \_\_ 225$$
$$225 = 225$$

Right

Side Lengths: 5, 7, 11

$$5^2 + 7^2 \_\_ 11^2$$
$$25 + 49 \_\_ 121$$
$$74 < 121$$

Obtuse

Side Lengths: $\sqrt{5}, 5, \sqrt{21}$

$$\left(\sqrt{5}\right)^2 + \left(\sqrt{21}\right)^2 \_\_ 5^2$$
$$5 + 21 \_\_ 25$$
$$26 > 25$$

Acute