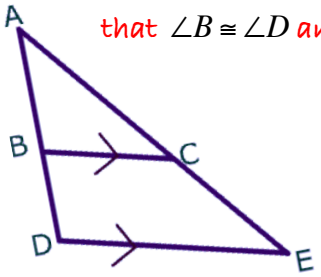
Triangle Theorems { 8.3 }

Secondary Math II Notes

OBJECTIVE: Use triangle similarity theorems to find missing values.

Is ΔABC similar to ΔADE? ΔABC and ΔADE share $\angle A$ so $\angle A \cong \angle A$. Also since $\overline{BC} \parallel \overline{DE}$, we know that $\angle B \cong \angle D$ and $\angle C \cong \angle E$ because they are corresponding angles. Thus ΔABC is similar to ΔADE by AA similarity.



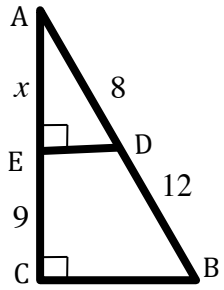
Since the two triangles are similar we know $\frac{AB}{AD} = \frac{AC}{AE}$.

Theorem 1: A line parallel to one side of a triangle divides the other two proportionally.

If ADE is any triangle and BC is drawn parallel to DE, then $\frac{AB}{BD} = \frac{AC}{CE}$

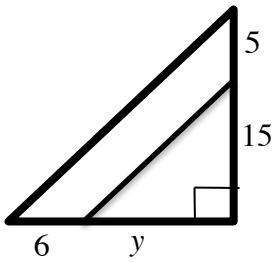
Find the missing value of the triangles below.

Assume $ED \parallel CB$



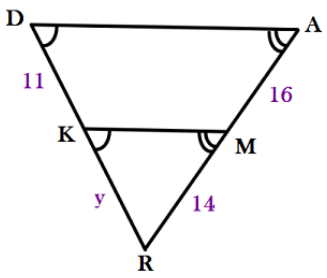
$\frac{x}{9} = \frac{8}{12} = \frac{2}{3}$ x=6

Assume $BD \parallel AE$



$\frac{6}{y} = \frac{5}{15} = \frac{1}{3}$ y=18

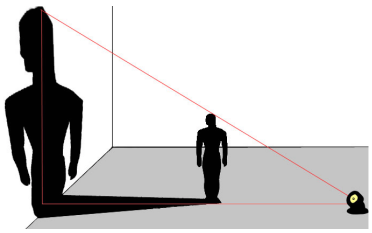
Assume $DA \parallel KM$



$\frac{11}{y} = \frac{16}{14} = \frac{8}{7}$ y=9.625

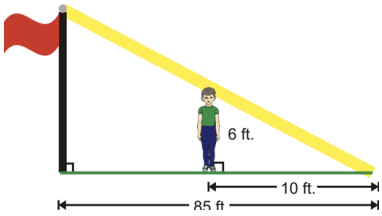
In the following image, how tall would the man be who is casting the shadow? Discuss how you could solve this. Using the picture to the right we can set up a triangle similarity proportion.

In this film the shadow is 50 feet tall and the distance from the projector to the man is 5 feet while the distance from the projector to the wall is 40 feet. How tall is the man?



$\frac{x}{50} = \frac{5}{40} = \frac{1}{8}$ x=6.25. The man is 6 feet 3 inches tall.

What if you wanted to measure the height of a flagpole using your friend George? He is 6 feet tall and his shadow is 10 feet long. At the same time, the shadow of the flagpole was 85 feet long. How tall is the flagpole?



$\frac{x}{85} = \frac{6}{10} = \frac{3}{5}$ x=51

Thus the flag pole is 51 ft. tall

On a sunny day, if a yardstick casts a 21-inch shadow, how tall is a building whose shadow is 168 feet?

Does it matter that we are given two different units? No because the ratios would still be the same.

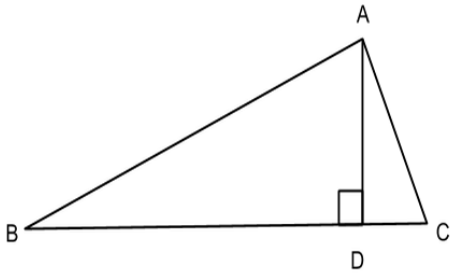
$\frac{168}{x} = \frac{21}{36} = \frac{7}{12}$ x=288. Thus the building is 288 feet tall.

Theorem 2: The altitude to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle.

$\Delta ABC \sim \Delta DBA \sim \Delta DAC$

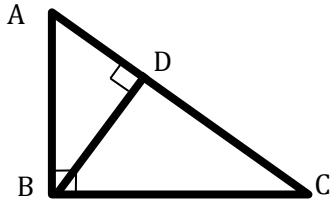
$\frac{AB}{DB} = \frac{AC}{DA} = \frac{BC}{BA}$ $\frac{DB}{DA} = \frac{DA}{DC} = \frac{BA}{AC}$

$\frac{AB}{DA} = \frac{AC}{DC} = \frac{BC}{AC}$

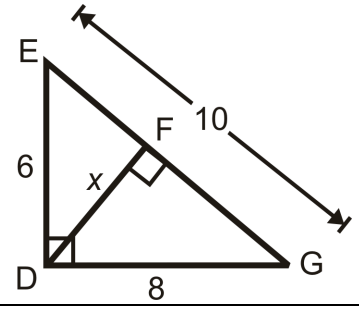


Identify the similar triangles in the figure below.

$\Delta ABC \sim \Delta BDA \sim \Delta CDB$

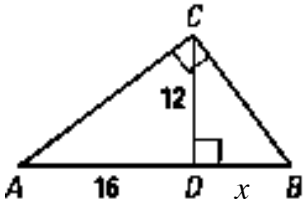


Find the missing value in the figure below.



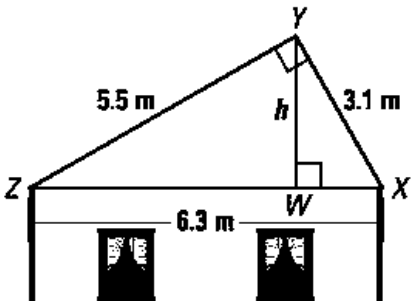
$\frac{6}{10} = \frac{x}{8}$, x=4.8

Find the missing value in the figure below.



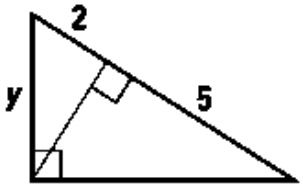
$\frac{12}{16} = \frac{x}{12}$ x=9

Find the height of the roof given the measurements on the figure below.



$\frac{5.5}{6.3} = \frac{h}{5.5}$ h=4.8. Thus the height of the roof is 4.8 in.

Find the missing value in the figure below.



$\frac{2}{y} = \frac{y}{(2+5)}$

$\frac{2}{y} = \frac{y}{7}$

$y^2 = 14, y = \sqrt{14}$